3.1 Exercises

1. (a) How is the number e defined?
   (b) Use a calculator to estimate the values of the limits
   $$\lim_{h \to 0} \frac{2.7^h - 1}{h} \quad \text{and} \quad \lim_{h \to 0} \frac{2.8^h - 1}{h}$$
   correct to two decimal places. What can you conclude about the value of e?

2. (a) Sketch, by hand, the graph of the function $f(x) = e^x$, paying particular attention to how the graph crosses the y-axis. What fact allows you to do this?
   (b) What types of functions are $f(x) = e^x$ and $g(x) = x^e$? Compare the differentiation formulas for f and g.
   (c) Which of the two functions in part (b) grows more rapidly when x is large?

3–28 □ Differentiate the function.

3. $f(x) = 5x - 1$
4. $F(x) = -4x^{10}$
5. $f(x) = x^2 + 3x - 4$
6. $g(x) = 5x^8 - 2x^5 + 6$
7. $y = x^{-2/5}$
8. $y = 5e^x + 3$
9. $V(r) = \frac{4}{3}\pi r^3$
10. $R(t) = 5t^{-3/5}$
11. $Y(t) = 6t^{-9}$
12. $R(x) = \dfrac{\sqrt{10}}{x^7}$
13. $F(x) = (16x)^3$
14. $y = \sqrt[3]{x}$
15. $g(x) = x^2 + \dfrac{1}{x^2}$
16. $f(t) = \sqrt{t} - \dfrac{1}{\sqrt{t}}$
17. $y = \dfrac{x^2 + 4x + 3}{\sqrt{x}}$
18. $y = \dfrac{x^2 - 2\sqrt{x}}{x}$
19. $y = 3x + 2e^x$
20. $y = \sqrt{x}\,(x - 1)$
21. $y = 4\pi^2$
22. $y = x^{4/3} - x^{2/3}$
23. $y = ax^2 + bx + c$
24. $y = A + \dfrac{B}{x} + \dfrac{C}{x^2}$
25. $y = x + \sqrt[5]{x^2}$
26. $u = \sqrt[3]{t^2} + 2\sqrt{t^3}$
27. $v = x\sqrt{x} + \dfrac{1}{x^2\sqrt{x}}$
28. $y = e^{x+1} + 1$

29–34 □ Find $f'(x)$. Compare the graphs of f and f' and use them to explain why your answer is reasonable.

29. $f(x) = 2x^2 - x^4$
30. $f(x) = 3x^5 - 20x^3 + 50x$
31. $f(x) = 3x^{15} - 5x^3 + 3$
32. $f(x) = x + \dfrac{1}{x}$
33. $f(x) = x - 3x^{1/3}$
34. $f(x) = x^2 + 2e^x$

35. (a) By zooming in on the graph of $f(x) = x^{2/5}$, estimate the value of $f'(2)$.
    (b) Use the Power Rule to find the exact value of $f'(2)$ and compare with your estimate in part (a).

36. (a) By zooming in on the graph of $f(x) = x^2 - 2e^x$, estimate the value of $f'(1)$.
    (b) Find the exact value of $f'(1)$ and compare with your estimate in part (a).

37–40 □ Find an equation of the tangent line to the curve at the given point. Illustrate by graphing the curve and the tangent line on the same screen.

37. $y = x + \dfrac{4}{x}$, $(2, 4)$
38. $y = x^{5/2}$, $(4, 32)$
39. $y = x + \sqrt{x}$, $(1, 2)$
40. $y = x^2 + 2e^x$, $(0, 2)$

41. (a) Use a graphing calculator or computer to graph the function $f(x) = x^4 - 3x^3 - 6x^2 + 7x + 30$ in the viewing rectangle $[-3, 5]$ by $[-10, 50]$.
    (b) Using the graph in part (a) to estimate slopes, make a rough sketch, by hand, of the graph of f'. (See Example 1 in Section 2.9.)
    (c) Calculate $f'(x)$ and use this expression, with a graphing device, to graph f'. Compare with your sketch in part (b).

42. (a) Use a graphing calculator or computer to graph the function $g(x) = e^x - 3x^2$ in the viewing rectangle $[-1, 4]$ by $[-8, 8]$.
    (b) Using the graph in part (a) to estimate slopes, make a rough sketch, by hand, of the graph of g'. (See Example 1 in Section 2.9.)
    (c) Calculate $g'(x)$ and use this expression, with a graphing device, to graph g'. Compare with your sketch in part (b).

43. Find the points on the curve $y = x^3 - x^2 - x + 1$ where the tangent is horizontal.

44. For what values of x does the graph of $f(x) = 2x^3 - 3x^2 - 6x + 87$ have a horizontal tangent?

45. Show that the curve $y = 6x^3 + 5x - 3$ has no tangent line with slope 4.

46. At what point on the curve $y = 1 + 2e^x - 3x$ is the tangent line parallel to the line $3x - y = 5$? Illustrate by graphing the curve and both lines.

47. Draw a diagram to show that there are two tangent lines to the parabola $y = x^2$ that pass through the point $(0, -4)$. Find the coordinates of the points where these tangent lines intersect the parabola.

3.2 Exercises

1. Find the derivative of $y = (x^2 + 1)(x^3 + 1)$ in two ways: by using the Product Rule and by performing the multiplication first. Do your answers agree?

2. Find the derivative of the function

$$F(x) = \frac{x - 3x\sqrt{x}}{\sqrt{x}}$$

in two ways: by using the Quotient Rule and by simplifying first. Show that your answers are equivalent. Which method do you prefer?

3–22 □ Differentiate.

Resources / Module 4 / Polynomial Models / Problems and Tests

3. $f(x) = x^2e^x$

4. $g(x) = \sqrt{x}\, e^x$

5. $y = \dfrac{e^x}{x^2}$

6. $y = \dfrac{e^x}{1 + x}$

7. $h(x) = \dfrac{x + 2}{x - 1}$

8. $f(u) = \dfrac{1 - u^2}{1 + u^2}$

9. $G(s) = (s^2 + s + 1)(s^2 + 2)$

10. $g(x) = (1 + \sqrt{x})(x - x^3)$

11. $H(x) = (x^3 - x + 1)(x^{-2} + 2x^{-3})$

12. $H(t) = e^t(1 + 3t^2 + 5t^4)$

13. $y = \dfrac{3t - 7}{t^2 + 5t - 4}$

14. $y = \dfrac{4t + 5}{2 - 3t}$

15. $y = \dfrac{x^2 + 4x + 3}{\sqrt{x}}$

16. $y = \dfrac{\sqrt{x} - 1}{\sqrt{x} + 1}$

17. $y = (r^2 - 2r)e^r$

18. $y = \dfrac{u^2 - u - 2}{u + 1}$

19. $y = \dfrac{1}{x^4 + x^2 + 1}$

20. $y = \dfrac{e^x}{x + e^x}$

21. $f(x) = \dfrac{x}{x + \dfrac{c}{x}}$

22. $f(x) = \dfrac{ax + b}{cx + d}$

23–26 □ Find an equation of the tangent line to the curve at the given point.

23. $y = \dfrac{2x}{x + 1}$, $(1, 1)$

24. $y = \dfrac{\sqrt{x}}{x + 1}$, $(4, 0.4)$

25. $y = 2xe^x$, $(0, 0)$

26. $y = \dfrac{e^x}{x}$, $(1, e)$

27. (a) The curve $y = 1/(1 + x^2)$ is called a **witch of Maria Agnesi**. Find an equation of the tangent line to this curve at the point $\left(-1, \frac{1}{2}\right)$.
(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

28. (a) The curve $y = x/(1 + x^2)$ is called a **serpentine**. Find an equation of the tangent line to this curve at the point $(3, 0.3)$.
(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

29. (a) If $f(x) = e^x/x^3$, find $f'(x)$.
(b) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f'.

30. (a) If $f(x) = x/(x^2 - 1)$, find $f'(x)$.
(b) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f'.

31. Suppose that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$, and $g'(5) = 2$. Find the values of (a) $(fg)'(5)$, (b) $(f/g)'(5)$, and (c) $(g/f)'(5)$.

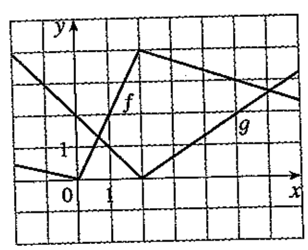
32. If $f(3) = 4$, $g(3) = 2$, $f'(3) = -6$, and $g'(3) = 5$, find the following numbers:
(a) $(f + g)'(3)$
(b) $(fg)'(3)$
(c) $(f/g)'(3)$
(d) $\left(\dfrac{f}{f - g}\right)'(3)$

33. If $f(x) = e^xg(x)$, where $g(0) = 2$ and $g'(0) = 5$, find $f'(0)$.

34. If $h(2) = 4$ and $h'(2) = -3$, find

$$\frac{d}{dx}\left(\frac{h(x)}{x}\right)\bigg|_{x=2}$$

35. If f and g are the functions whose graphs are shown, let $u(x) = f(x)g(x)$ and $v(x) = f(x)/g(x)$.
(a) Find $u'(1)$.
(b) Find $v'(5)$.



36. If f is a differentiable function, find an expression for the derivative of each of the following functions.
(a) $y = x^2f(x)$
(b) $y = \dfrac{f(x)}{x^2}$
(c) $y = \dfrac{x^2}{f(x)}$
(d) $y = \dfrac{1 + xf(x)}{\sqrt{x}}$

37. In this exercise we estimate the rate at which the total personal income is rising in the Miami–Ft. Lauderdale metropolitan area. In July, 1993, the population of this area was 3,354,000, and the population was increasing at roughly 45,000 people per year. The average annual income was \$21,107 per capita, and this average was increasing at about \$1900 per year (well above the national average of about \$660 yearly). Use the

by time t, then the derivative dp/dt represents the rate of spread of the rumor (see Exercise 68 in Section 3.5).

Summary

Velocity, density, current, power, and temperature gradient in physics, rate of reaction and compressibility in chemistry, rate of growth and blood velocity gradient in biology, marginal cost and marginal profit in economics, rate of heat flow in geology, rate of improvement of performance in psychology, rate of spread of a rumor in sociology—these are all special cases of a single mathematical concept, the derivative.

This is an illustration of the fact that part of the power of mathematics lies in its abstractness. A single abstract mathematical concept (such as the derivative) can have different interpretations in each of the sciences. When we develop the properties of the mathematical concept once and for all, we can then turn around and apply these results to all of the sciences. This is much more efficient than developing properties of special concepts in each separate science. The French mathematician Joseph Fourier (1768–1830) put it succinctly: "Mathematics compares the most diverse phenomena and discovers the secret analogies that unite them."

3.3 Exercises

1–6 □ A particle moves according to a law of motion $s = f(t)$, $t \geqslant 0$, where t is measured in seconds and s in feet.

(a) Find the velocity at time t.
(b) What is the velocity after 3 s?
(c) When is the particle at rest?
(d) When is the particle moving in the positive direction?
(e) Find the total distance traveled during the first 8 s.
(f) Draw a diagram like Figure 2 to illustrate the motion of the particle.

1. $f(t) = t^2 - 10t + 12$

2. $f(t) = t^3 - 9t^2 + 15t + 10$

3. $f(t) = t^3 - 12t^2 + 36t$

4. $f(t) = t^4 - 4t + 1$

5. $s = \dfrac{t}{t^2 + 1}$

6. $s = \sqrt{t}\,(3t^2 - 35t + 90)$

7. The position function of a particle is given by

$$s = t^3 - 4.5t^2 - 7t \qquad t \geqslant 0$$

When does the particle reach a velocity of 5 m/s?

8. If a ball is thrown vertically upward with a velocity of 80 ft/s, then its height after t seconds is $s = 80t - 16t^2$.
(a) What is the maximum height reached by the ball?
(b) What is the velocity of the ball when it is 96 ft above the ground on its way up? On its way down?

9. (a) A company makes computer chips from square wafers of silicon. It wants to keep the side length of a wafer very close to 15 mm and it wants to know how the area $A(x)$ of a wafer changes when the side length x changes. Find $A'(15)$ and explain its meaning in this situation.
(b) Show that the rate of change of the area of a square with respect to its side length is half its perimeter. Try to explain geometrically why this is true by drawing a square whose side length x is increased by an amount Δx. How can you approximate the resulting change in area ΔA if Δx is small?

10. (a) Sodium chlorate crystals are easy to grow in the shape of cubes by allowing a solution of water and sodium chlorate to evaporate slowly. If V is the volume of such a cube with side length x, calculate dV/dx when $x = 3$ mm and explain its meaning.
(b) Show that the rate of change of the volume of a cube with respect to its edge length is equal to half the surface area of the cube. Explain geometrically why this result is true by arguing by analogy with Exercise 9(b).

11. (a) Find the average rate of change of the area of a circle with respect to its radius r as r changes from
(i) 2 to 3 (ii) 2 to 2.5 (iii) 2 to 2.1
(b) Find the instantaneous rate of change when $r = 2$.
(c) Show that the rate of change of the area of a circle with respect to its radius (at any r) is equal to the circumference of the circle. Try to explain geometrically why this is true by drawing a circle whose radius is increased by an amount Δr. How can you approximate the resulting change in area ΔA if Δr is small?

12. A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of 60 cm/s. Find the rate at which the area within the circle is increasing after (a) 1 s, (b) 3 s, and (c) 5 s. What can you conclude?

So if we denote by ε the difference between the difference quotient and the derivative, we obtain

$$\lim_{\Delta x \to 0} \varepsilon = \lim_{\Delta x \to 0} \left(\frac{\Delta y}{\Delta x} - f'(a) \right) = f'(a) - f'(a) = 0$$

But
$$\varepsilon = \frac{\Delta y}{\Delta x} - f'(a) \quad \Rightarrow \quad \Delta y = f'(a)\,\Delta x + \varepsilon\,\Delta x$$

Thus, for a differentiable function f, we can write

$$\boxed{7} \qquad \Delta y = f'(a)\,\Delta x + \varepsilon\,\Delta x \qquad \text{where } \varepsilon \to 0 \text{ as } \Delta x \to 0$$

This property of differentiable functions is what enables us to prove the Chain Rule.

Proof of the Chain Rule Suppose $u = g(x)$ is differentiable at a and $y = f(u)$ is differentiable at $b = g(a)$. If Δx is an increment in x and Δu and Δy are the corresponding increments in u and y, then we can use Equation 7 to write

$$\boxed{8} \qquad \Delta u = g'(a)\,\Delta x + \varepsilon_1\,\Delta x = [g'(a) + \varepsilon_1]\,\Delta x$$

where $\varepsilon_1 \to 0$ as $\Delta x \to 0$. Similarly

$$\boxed{9} \qquad \Delta y = f'(b)\,\Delta u + \varepsilon_2\,\Delta u = [f'(b) + \varepsilon_2]\,\Delta u$$

where $\varepsilon_2 \to 0$ as $\Delta u \to 0$. If we now substitute the expression for Δu from Equation 8 into Equation 9, we get

$$\Delta y = [f'(b) + \varepsilon_2][g'(a) + \varepsilon_1]\,\Delta x$$

so
$$\frac{\Delta y}{\Delta x} = [f'(b) + \varepsilon_2][g'(a) + \varepsilon_1]$$

As $\Delta x \to 0$, Equation 8 shows that $\Delta u \to 0$. So both $\varepsilon_1 \to 0$ and $\varepsilon_2 \to 0$ as $\Delta x \to 0$. Therefore

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} [f'(b) + \varepsilon_2][g'(a) + \varepsilon_1]$$
$$= f'(b)g'(a) = f'(g(a))g'(a)$$

This proves the Chain Rule. □

3.5 Exercises

1–6 □ Write the composite function in the form $f(g(x))$. [Identify the inner function $u = g(x)$ and the outer function $y = f(u)$.] Then find the derivative dy/dx.

Resources / Module 4 / Trigonometric Models / Chain Rule Practice

1. $y = (x^2 + 4x + 6)^5$
2. $y = \tan 3x$
3. $y = \cos(\tan x)$
4. $y = \sqrt[3]{1 + x^3}$
5. $y = e^{\sqrt{x}}$
6. $y = \sin(e^x)$

7–42 □ Find the derivative of the function.

7. $F(x) = (x^3 + 4x)^7$
8. $F(x) = (x^2 - x + 1)^3$
9. $g(x) = \sqrt{x^2 - 7x}$
10. $f(t) = \dfrac{1}{(t^2 - 2t - 5)^4}$
11. $h(t) = \left(t - \dfrac{1}{t}\right)^{3/2}$
12. $f(t) = \sqrt[3]{1 + \tan t}$

13. $y = \cos(a^3 + x^3)$

14. $y = a^3 + \cos^3 x$

15. $y = e^{-mx}$

16. $y = 4 \sec 5x$

17. $G(x) = (3x - 2)^{10}(5x^2 - x + 1)^{12}$

18. $g(t) = (6t^2 + 5)^3(t^3 - 7)^4$

19. $y = (2x - 5)^4(8x^2 - 5)^{-3}$

20. $y = (x^2 + 1)\sqrt[3]{x^2 + 2}$

21. $y = xe^{-x^2}$

22. $y = e^{-5x} \cos 3x$

23. $F(y) = \left(\dfrac{y - 6}{y + 7}\right)^3$

24. $s(t) = \sqrt[4]{\dfrac{t^3 + 1}{t^3 - 1}}$

25. $f(z) = \dfrac{1}{\sqrt[5]{2z - 1}}$

26. $f(x) = \dfrac{x}{\sqrt{7 - 3x}}$

27. $y = \tan(\cos x)$

28. $y = \dfrac{\sin^2 x}{\cos x}$

29. $y = 5^{-1/x}$

30. $y = \sqrt{1 + 2 \tan x}$

31. $y = \sin^3 x + \cos^3 x$

32. $y = \sin^2(\cos kx)$

33. $y = (1 + \cos^2 x)^6$

34. $y = x \sin \dfrac{1}{x}$

35. $y = \dfrac{e^{3x}}{1 + e^x}$

36. $y = e^{5 \sin \theta}$

37. $y = e^{x \cos x}$

38. $y = \sin(\sin(\sin x))$

39. $y = \sqrt{x + \sqrt{x}}$

40. $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

41. $y = \sin\left(\tan \sqrt{\sin x}\right)$

42. $y = 2^{3^{x^2}}$

43–46 □ Find an equation of the tangent line to the curve at the given point.

43. $y = \dfrac{8}{\sqrt{4 + 3x}}$, $(4, 2)$

44. $y = \sin x + \cos 2x$, $(\pi/6, 1)$

45. $y = \sin(\sin x)$, $(\pi, 0)$

46. $y = 10^x$, $(1, 10)$

47. (a) Find an equation of the tangent line to the curve $y = 2/(1 + e^{-x})$ at the point $(0, 1)$.
(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

48. (a) The curve $y = |x|/\sqrt{2 - x^2}$ is called a **bullet-nose curve**. Find an equation of the tangent line to this curve at the point $(1, 1)$.
(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

49. (a) If $f(x) = \sqrt{1 - x^2}/x$, find $f'(x)$.
(b) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f'.

50. (a) If $f(x) = 1/(\cos^2 \pi x + 9 \sin^2 \pi x)$, find $f'(x)$.
(b) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f'.

51. Find all points on the graph of the function $f(x) = 2 \sin x + \sin^2 x$ at which the tangent line is horizontal.

52. Find the x-coordinates of all points on the curve $y = \sin 2x - 2 \sin x$ at which the tangent line is horizontal.

53. Suppose that $F(x) = f(g(x))$ and $g(3) = 6$, $g'(3) = 4$, $f'(3) = 2$, and $f'(6) = 7$. Find $F'(3)$.

54. Suppose that $w = u \circ v$ and $u(0) = 1$, $v(0) = 2$, $u'(0) = 3$, $u'(2) = 4$, $v'(0) = 5$, and $v'(2) = 6$. Find $w'(0)$.

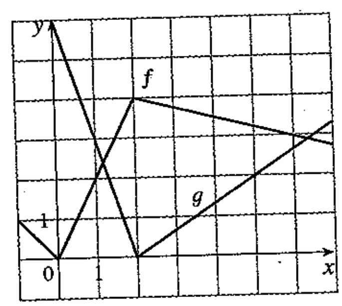
55. A table of values for f, g, f', and g' is given.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

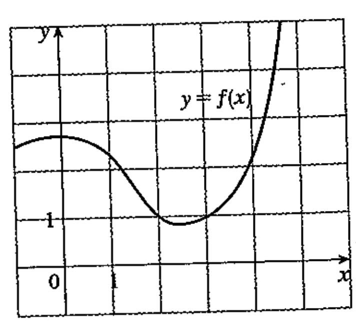
(a) If $h(x) = f(g(x))$, find $h'(1)$.
(b) If $H(x) = g(f(x))$, find $H'(1)$.

56. Let f and g be the functions in Exercise 55.
(a) If $F(x) = f(f(x))$, find $F'(2)$.
(b) If $G(x) = g(g(x))$, find $G'(3)$.

57. If f and g are the functions whose graphs are shown, let $u(x) = f(g(x))$, $v(x) = g(f(x))$, and $w(x) = g(g(x))$. Find each derivative, if it exists. If it does not exist, explain why.
(a) $u'(1)$ (b) $v'(1)$ (c) $w'(1)$



58. If f is the function whose graph is shown, let $h(x) = f(f(x))$ and $g(x) = f(x^2)$. Use the graph of f to estimate the value of each derivative.
(a) $h'(2)$ (b) $g'(2)$



$$D^4 \cos x = \cos x$$

$$D^5 \cos x = -\sin x$$

We see that the successive derivatives occur in a cycle of length 4 and, in particular, $D^n \cos x = \cos x$ whenever n is a multiple of 4. Therefore

$$D^{24} \cos x = \cos x$$

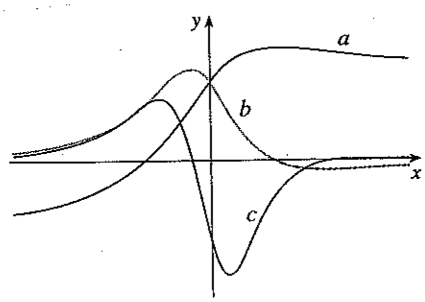
and, differentiating three more times, we have

$$D^{27} \cos x = \sin x$$ □

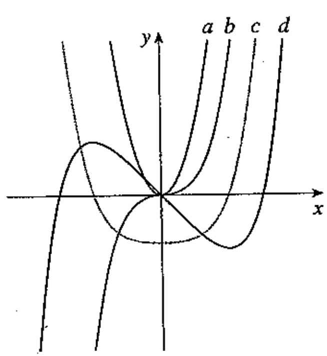
We have seen that one application of second and third derivatives occurs in analyzing the motion of objects using acceleration and jerk. We will investigate another application of second derivatives in Exercise 62 and in Section 4.3, where we show how knowledge of f'' gives us information about the shape of the graph of f. In Chapter 11 we will see how second and higher derivatives enable us to represent functions as sums of infinite series.

3.7 Exercises

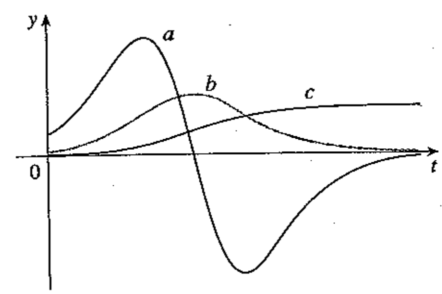
1. The figure shows the graphs of f, f', and f''. Identify each curve, and explain your choices.



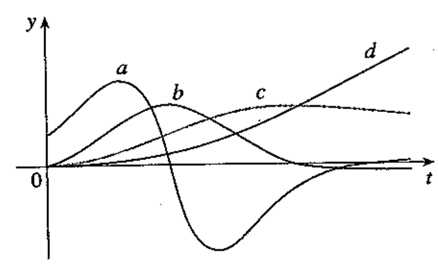
2. The figure shows graphs of f, f', f'', and f'''. Identify each curve, and explain your choices.



3. The figure shows the graphs of three functions. One is the position function of a car, one is the velocity of the car, and one is its acceleration. Identify each curve, and explain your choices.



4. The figure shows the graphs of four functions. One is the position function of a car, one is the velocity of the car, one is its acceleration, and one is its jerk. Identify each curve, and explain your choices.



5–20 □ Find the first and second derivatives of the function.

5. $f(x) = x^5 + 6x^2 - 7x$

6. $f(t) = t^8 - 7t^6 + 2t^4$

7. $y = \cos 2\theta$

8. $y = \theta \sin \theta$

9. $h(x) = \sqrt{x^2 + 1}$

10. $G(r) = \sqrt{r} + \sqrt[3]{r}$

11. $F(s) = (3s + 5)^8$

12. $g(u) = \dfrac{1}{\sqrt{1 - u}}$

13. $y = \dfrac{x}{1-x}$

14. $y = xe^{cx}$

15. $y = (1 - x^2)^{3/4}$

16. $y = \dfrac{x^2}{x+1}$

17. $H(t) = \tan 3t$

18. $g(s) = s^2 \cos s$

19. $g(t) = t^3 e^{5t}$

20. $h(x) = \tan^{-1}(x^2)$

21. (a) If $f(x) = 2\cos x + \sin^2 x$, find $f'(x)$ and $f''(x)$.
(b) Check to see that your answers to part (a) are reasonable by comparing the graphs of f, f', and f''.

22. (a) If $f(x) = e^x - x^3$, find $f'(x)$ and $f''(x)$.
(b) Check to see that your answers to part (a) are reasonable by comparing the graphs of f, f', and f''.

23–24 □ Find y'''.

23. $y = \sqrt{2x+3}$

24. $y = \dfrac{1-x}{1+x}$

25. If $f(x) = (2 - 3x)^{-1/2}$, find $f(0)$, $f'(0)$, $f''(0)$, and $f'''(0)$.

26. If $g(t) = (2 - t^2)^6$, find $g(0)$, $g'(0)$, $g''(0)$, and $g'''(0)$.

27. If $f(\theta) = \cot\theta$, find $f'''(\pi/6)$.

28. If $g(x) = \sec x$, find $g'''(\pi/4)$.

29–32 □ Find y'' by implicit differentiation.

29. $x^3 + y^3 = 1$

30. $\sqrt{x} + \sqrt{y} = 1$

31. $x^2 + xy + y^2 = 1$

32. $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

33–37 □ Find a formula for $f^{(n)}(x)$.

33. $f(x) = x^n$

34. $f(x) = \dfrac{1}{(1-x)^2}$

35. $f(x) = e^{2x}$

36. $f(x) = \sqrt{x}$

37. $f(x) = \dfrac{1}{3x^3}$

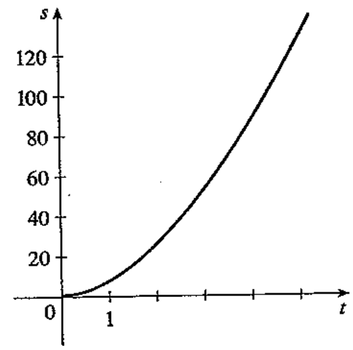
38–40 □ Find the given derivative by finding the first few derivatives and observing the pattern that occurs.

38. $D^{99} \sin x$

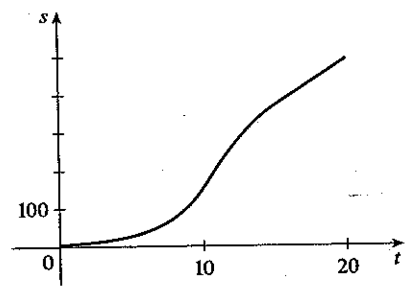
39. $D^{50} \cos 2x$

40. $D^{1000} xe^{-x}$

41. A car starts from rest and the graph of its position function is shown in the figure, where s is measured in feet and t in seconds. Use it to graph the velocity and estimate the acceleration at $t = 2$ seconds from the velocity graph. Then sketch a graph of the acceleration function.



42. (a) The graph of a position function of a car is shown, where s is measured in feet and t in seconds. Use it to graph the velocity and acceleration of the car. What is the acceleration at $t = 10$ seconds?



(b) Use the acceleration curve from part (a) to estimate the jerk at $t = 10$ seconds. What are the units for jerk?

43–46 □ The equation of motion is given for a particle, where s is in meters and t is in seconds. Find (a) the velocity and acceleration as functions of t, (b) the acceleration after 1 second, and (c) the acceleration at the instants when the velocity is 0.

43. $s = t^3 - 3t$

44. $s = t^2 - t + 1$

45. $s = \sin 2\pi t$

46. $s = 2t^3 - 7t^2 + 4t + 1$

47–48 □ An equation of motion is given, where s is in meters and t in seconds. Find (a) the times at which the acceleration is 0 and (b) the displacement and velocity at these times.

47. $s = t^4 - 4t^3 + 2$

48. $s = 2t^3 - 9t^2$